

12.6 Probability versus Determinism

Physics Tool box

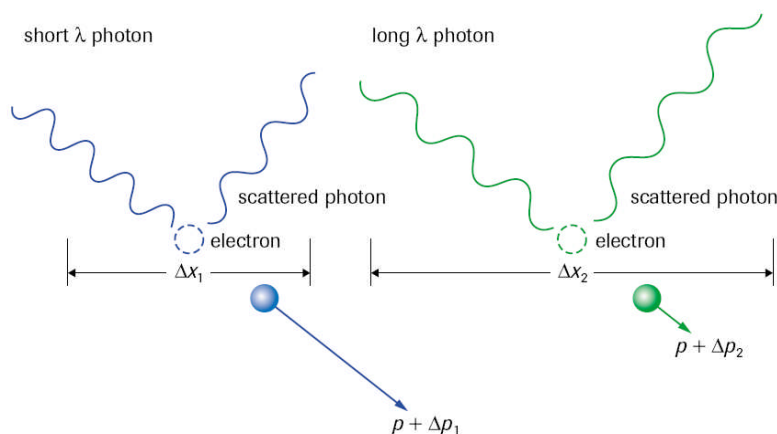
- In classical mechanics, the objects we identify as particles always behave like particles, while wave phenomena always exhibit pure wave properties.
- In quantum mechanics, light is composed of photons possessing distinct particle characteristics, and electrons behave like waves with a definite wavelength.
- Heisenberg proposed that an inherent uncertainty exists in the simultaneous determination of any measured quantity due to the quantum-mechanical wave aspect of particles.
- We are unable to measure both the position and the momentum of the electron with unlimited accuracy. Heisenberg was able to determine the limits of these inherent uncertainties and to express them mathematically as

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$

Once scientists entered the new and uncharted world of quantum mechanics, many phenomena were discovered that were both strange and difficult to visualize. They were used to the Newtonian world (classical) where objects that were identified as particles always behaved like particles, and wave phenomena always exhibited pure wave properties. But the quantum hypothesis of Planck, Einstein, and Bohr created a new dilemma: light that has been traditionally been viewed as a wave was now thought of as photons, particles possessing distinct particle characteristics. While electrons, thought of as tiny particles with a definite mass and charge, behaved like waves with a definite wavelength. Clearly a new way of looking at wave and particle phenomena was needed.

A German physicist, Werner Heisenberg proposed that there was also another problem in accurately determining certain measured properties that scientists were investigating. He proposed that there was always some inherent uncertainty in the determination of an object's exact location and its exact speed and energy (momentum).

Imagine that we are trying to determine the position of an electron within an atom. To do this, we bombard the atom with highly energetic electromagnetic radiation of a given wavelength. A photon of wavelength λ will have a momentum $\frac{h}{\lambda}$.



We also know from the Compton effect that this photon will transfer some of its momentum to the electron when they interact. Thus the electron will acquire a new and unknown value of momentum. Now the more accurately we try to locate the electron by using a smaller wavelength, the less precisely we know its momentum (since we have kicked it more). To provide less momentum, we need a longer wavelength, but then we become less accurate in location its precise position. Heisenberg was able to determine the limits of these inherent uncertainties via the Heisenberg Uncertainty Principle:

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$

Example

For a large mass of 1.0kg assume we can determine its position with an accuracy of $\pm 10^{-7} m$. What is the maximum accuracy of the speed we can expect to obtain?

Solution:

$$\begin{aligned} \Delta x \Delta p &\geq \frac{h}{2\pi} \\ \Delta x m \Delta v &\geq \frac{h}{2\pi} \\ \Delta x \Delta v &\geq \frac{h}{2\pi m} \\ &\geq \frac{6.63 \times 10^{-34} J \cdot s}{2\pi (1.0 kg)} \\ &\geq 1.1 \times 10^{-34} \frac{J \cdot s}{kg} \\ \Delta v &\geq \frac{1.1 \times 10^{-34} \frac{J \cdot s}{kg}}{10^{-7} m} \\ &\geq 10^{-27} \frac{m}{s} \end{aligned}$$

Thus the uncertainty of Δv is $\pm 10^{-27} \frac{m}{s}$

This demonstrates that such small uncertainties do not show up on large macroscopic objects, but on the other hand if the mass of the particle was that of an electron $9.1 \times 10^{-31} kg$ then

$$\begin{aligned}
 \Delta x \Delta v &\geq \frac{h}{2\pi m} \\
 &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi (9.1 \times 10^{-31} \text{ kg})} \\
 &\geq 1.2 \times 10^{-4} \frac{\text{J} \cdot \text{s}}{\text{kg}} \\
 \Delta v &\geq \frac{1.2 \times 10^{-4} \frac{\text{J} \cdot \text{s}}{\text{kg}}}{10^{-7} \text{ m}} \\
 &\geq 10^3 \frac{\text{m}}{\text{s}}
 \end{aligned}$$

We cannot know its speed within an uncertainty of less than $\pm 10^3 \frac{\text{m}}{\text{s}}$